Reaction Wheel Actuation for Improving Planar Biped Walking Efficiency

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Abstract—Reaction wheel systems (RWSs) are beneficial in improving bipedal walking stability, and theoretical work has shown that they can also improve the efficiency of preplanned walking motions. This work provides the first demonstration of RWS-related efficiency improvements on a physical robot and identifies the energy-saving paradigm. Gaits for a five-link planar biped are generated via trajectory optimization with and without reaction wheels. Comparisons of the resulting system behavior show a 5-10% efficiency advantage for robots with an RWS under typical walking conditions and a larger advantage near regions of marginal dynamic feasibility. The savings is mainly accomplished by enabling trajectories with better joint motor utilization, not by minimizing impact losses or acting as a kinetic energy recovery system. The RWS also improves centroidal angular momentum regulation, with the reduction in centroidal angular momentum amplitude reaching 30% at high speeds. Simulations and experiments with a five-link planar biped validate the efficiency improvements seen in optimization. This demonstrates the viability of using efficient inertial actuators to improve the efficiency of complex walking robots.

I. INTRODUCTION

Mechanically speaking, stabilization of bipedal systems generally follows one of three strategies: relocation of a foot (stepping), generation of ankle torque, and inertial stabilization through body link acceleration [1], all of which have been observed in humans [2]. Stepping is well described by the inverted pendulum model, leading to foot placement strategies such as the capture point concept [3], and ankle torque stabilization has been well developed within the context of the zero moment point [4]. Inertial stabilization has made recent progress with force control strategies [5]–[7]. Nevertheless, the efficacy and potential of inertial strategies are difficult to assess due to the complexity of, and mechanical differences between, various biped models. To simplify the analysis of upper-body inertial control, a lower order model consisting of a single rotating reaction wheel is common [1], [3], [8]–[10]. Similar to the inverted pendulum model of walking, this provides more insight into the underlying dynamics and allows for more straightforward comparisons across systems.

Beyond the use of reaction wheels as modeling tools, some work has examined using reaction wheel stabilizer (RWS) systems or closely related control moment gyroscopes (CMGs) for stabilizing bipeds. Such systems are common in satellite attitude control [11] and have been suggested as aids for human balance [12]. While mathematically similar in effect to upper-body link motion (minus linear momentum), reaction wheels are less constrained by joint limits and, therefore, represent the upper feasibility limit with angular-momentum-based inertial actuation. Furthermore, dedicated reaction wheel hardware does not rely on the motion of upper body links that may be reserved for other tasks. Simulation work has shown that CMGs can stabilize walking bipeds [13], and RWS stabilization has been realized in a physical prototype [14]. Simulation of a passive walker showed a significant improvement in the range of stable gaits with an RWS [15]. The authors’ prior work [16] showed that use of an RWS can extend the range of feasible periodic gaits and decrease energy consumption in simulation, particularly when operating at non-ideal speeds and step lengths.

Advanced control strategies such as prioritized task space control [7] enable complex robots to take advantage of multiple links to exert commanded torques and forces on the body. Such methods, however, yield little insight into the relationship between upper-body design and gait efficiency or disturbance response potential, nor do they provide methods to determine what motions should be followed at a given instant. Indeed, lower order models or templates continue to be important for both planning motions and understanding the underlying physics of a system [17]. Whether or not the target robot includes an RWS actuator, an understanding of what inertial actuation can accomplish and how angular momentum should be managed is important for biped design and control. This work seeks to understand the connection between a robot’s ability to store/release angular momentum and its efficiency during walking. This is done by generating energy-optimal walking gaits under a variety of conditions for a planar five-link biped model based on the ERNIE robot [18] shown in
Fig. 1 This extends work with a similar model [16] to include a larger range of operation, thorough analysis of the resulting gaits, and validation via simulation and experiment.

II. NONLINEAR OPTIMIZATION FOR MOTION GENERATION

To compare bipeds with and without an RWS in terms of the energetic efficiency and dynamic feasibility of walking, dynamically feasible trajectories for both are generated over a range of conditions. Global optimal motion generation facilitates a valid comparison by identifying the best possible task-satisfying motions for each. Specifically in this work, a direct transcription approach is used because it works well with high dimensional systems and leverages existing NLP solvers such as SNOPT [19] and IPOPT [20].

A. Optimization Formulation

A five-link, planar biped model based on the underactuated biped ERNIE [15] is used to represent the sagittal plane dynamics of a walking robot. Physical parameters of the model are listed in Table I. The model takes the form

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = B(q)u + J(q)\lambda, \]  

where \( q \) is a set of generalized coordinates, \( \dot{q} \) is a set of system velocities, \( M \) is the mass matrix, \( C \) is the Coriolis matrix, \( G \) is the gravity vector, \( B \) is the matrix that maps control inputs \( (u) \) to generalized forces, and \( J \) is the contact Jacobian that maps contact forces \( (\lambda) \) to generalized forces. The base model consists of seven generalized coordinates. Three define the pose of the body in planar space, and four define the hip and knee joint angles. Including an RWS adds an eighth coordinate for the reaction wheel position.

A system trajectory \( X \) is encoded by a time-series of \( N \) waypoints, each consisting of generalized coordinates, generalized velocities, control inputs, and contact forces. Waypoints are separated by time steps of length \( h \), where \( h \) is an optimization variable. The equations of motion in (1) can then be rewritten as a nonlinear constraint equation using a finite difference approximation for the coordinate space acceleration.

\[ q_i - q_{i-1} - h\dot{q}_{i} = 0 \]  

(2)

\[ M_i[\ddot{q}_i - \dot{q}_{i-1}] + h[C_i\dot{q}_i + G_i(q_i) - B_iu_i - J_i\lambda_i] = 0, \]  

(3)

where \( i \) denotes the \( i \)th waypoint in the series. In addition to dynamic motion constraints, a set of contact constraints, including non-penetration, no-slip, friction cone, positive normal force, and non-flight constraints, were applied to enforce a perfectly dissipative unidirectional ground contact model. Additional details are in [21].

As detailed in [16] and [21], a ground clearance constraint ensures sufficient clearance between the swing foot and the ground over 80% of the step (permitting double support in the first 20%). This avoids unintentional ground contacts resulting from joint tracking errors and measurement noise in hardware. Experimental experience with ERNIE previously found improved experimental success with a touchdown-approach constraint limiting the change in swing shank angle in the last 20% of the gait [22]. Recent work, however, has achieved superior reliability by constraining the horizontal swing foot velocity to be near zero during the same phase of gait.

\[ |J_i\dot{q}_{i,x}| < 0.01 \quad i/N \geq 0.8. \]  

(4)

To ensure physically realistic results and improve optimization convergence, control torques, step length, step period, and forward velocity are required to be within reasonable values, and system coordinates, coordinate velocities, joint angles, and joint velocities are limited to match the capabilities of the experimental system. These limiting values have generally been set loose enough to avoid interference with typical walking motions and are therefore mostly inactive. This includes a generous \( \pm 0.2 \) rad postural constraint for the torso that is examined in Section III-B. The one exception is that the knees are required to maintain at least 5 degrees of flexion to simulate the bending stance common to many walking robots. This also causes the specific cost of transport (CoT) to be insensitive to added mass in the torso [21], emulating similar energetics observed in human walking [23]. Nevertheless, to guarantee that differences in mass don’t interfere with efficiency comparisons, the base model is loaded with a dead weight equal to the weight of the RWS.

B. Objective Function

Energetic efficiency, as measured by CoT, is the objective function in this work. CoT is computed by normalizing the energetic cost \( E \) per unit distance traveled \( d \) by the weight of the robot \( mg \). A standard DC motor model shows that motor power \( P_m \), which is integrated to obtain total energy cost, is a sum of mechanical work and ohmic loss.

\[ P_m = \eta \omega \tau + \frac{1}{K_m^2} \tau^2; \]  

(5)

where \( \omega \) is the motor speed, \( \tau \) is the motor torque, \( \eta \) is the gear train efficiency, and \( K_m \) is the motor constant, which generally correlates with motor size. The ohmic loss coefficient \( K_\omega \), a measure of the energetic cost of generating torque, is defined as \( \frac{1}{K_m^2} \), and its value has a significant effect on the energetic performance and all derived results. As such, a survey of existing legged robots was performed, resulting in the selection of \( K_\omega = 0.1 \) as being representative of current and anticipated legged robots with low impedance drive systems [21]. Efficiency \( \eta \) was set at 100% for positive joint work and 0% for negative joint work to model the poor regenerative efficiency of most geared servo motors. The regenerative efficiency of the RWS was set high (80%) to match the expected performance of a direct drive RWS.
Fig. 2: Optimization cost of transport (CoT) over the operational range of the base model. The red line indicates the lowest cost gait at each speed. Dotted lines indicate lines of constant step period. Gaits with periods longer than 1.5 s are not shown.

III. OPTIMIZATION RESULTS

The results in [16] showed that the behavior of the RWS changed with both speed and step length, indicating a need to study the full operational range. As such, a large family of walking gaits was generated over a grid of speeds and step lengths. These quantities have been non-dimensionalized to better accommodate comparison with other legged systems.

\[ NS = \frac{v}{\sqrt{gl}} \quad \text{and} \quad NSL = \frac{d}{l}, \]  

(6)

where NS stands for normalized speed, NSL stands for normalized step length, \( v \) is the walking speed, \( g \) is gravity, and \( l \) is the leg length from hip to foot. Figure 2 shows the resulting energy cost surface for the base model. The left side of the map terminates at a step period of 1.5 seconds. Gaits with periods longer than this exist, but are of little value since they involve prolonged periods of quasi-static balancing. The right side of the map terminates at dynamic feasibility boundaries. The upper boundary corresponds very closely to the “pendular running” transition found in [24]. The lower boundary is a result of friction constraints, with gaits here showing extended periods of maximum friction utilization.

This process can be repeated with the RWS-utilizing model to generate a similar operational efficiency map. Comparing these maps leads to the efficiency improvement map shown in Figure 3. Computing the ratio of two optimization results amplifies the noise, but important trends are visible. First, the RWS improves walking efficiency over the entire operating range of the base model. Second, the RWS expands the domain of feasible operating points (shown in blue). At lower speeds, the RWS has a small effect, but over the majority of the operating range, it leads to a 5–10% efficiency gain. Near areas of marginal feasibility, the efficiency gains become large. The complex dynamics of a biped make it difficult to intuitively understand the behavior of the RWS. The following analysis seeks to establish the efficiency improvement mechanism by evaluating three proposed hypotheses.

A. ESR Hypothesis

The first hypothesis states that the RWS acts as an energy storage and return (ESR) system, removing kinetic energy at certain parts of the gait and efficiently restoring that energy when advantageous. This hypothesis is evaluated by examining the energy exchange in walking gaits. Initial analysis shows that the magnitude of RWS work is relatively large. A distinction must be made, however, between work done by the RWS on the rest of the body and work done to accelerate the reaction wheel. The latter, which amounts to ‘spin-up’ kinetic energy, is a bi-product of the torque generation process and can be scaled arbitrarily by resizing the reaction wheel. This makes it useful only in evaluating the efficiency of the RWS itself, with larger reaction wheels storing less energy and consequently being more efficient. Comparisons must therefore be restricted to the work done by the RWS on the body. In this light, the RWS’s energetic contributions are very small compared to the rest of the joints. Moreover, the work performed by the RWS is overwhelmingly positive, which is not characteristic of an ESR system. Taken together, these facts weaken the argument that the RWS can be characterized as an ESR system.
B. Push-off Hypothesis

The second hypothesis states that the RWS stores momentum shortly before touchdown and restores it afterward, altering the energy dissipation of touchdown in much the same way as ankle plantar flexion at “push-off” does. By comparing the system’s kinetic energy before and after impact, impact losses were calculated for the base and RWS-utilizing models across a range of speeds (Figure 4). The RWS results in almost no change in impact loss for all speeds, directly refuting the push-off hypothesis. In these walking gaits, the torso is positioned slightly forward of the hip prior to touchdown and allowed to fall forward during touchdown. This is similar to the approach for impact loss reduction in [25], suggesting that the RWS may be redundant with the torso for the purposes of impact loss reduction. To test this, a series of gaits were generated for both models with strict vertical postural constraints on the torso. Contrary to expectations, the torso constraint significantly reduced impact losses (red line in Figure 4), but more so for the base model than the RWS-utilizing model. Comparing changes in CoT (Figure 5) reveals that removing all torso freedom increases walking cost of the base model by 10-30% over the useful speed range. As in the unconstrained case, use of the RWS reduces the cost of walking, but can’t fully compensate for the locked torso at most speeds. Since the RWS is more than capable of replicating the angular component of torso momentum, this suggests that the efficiency loss due to the torso constraint and the RWS-based efficiency gains involve separate mechanisms.

This demonstrates the danger of using impact loss as a surrogate for energy cost. In extremely simple models, impact loss can be effectively eliminated by introducing a tuned spring between the legs and either a torso [25] or a reaction wheel [25]. Nevertheless, more realistic models, such as the one herein, demonstrate that such elimination is more costly than allowing the impact. To accommodate the torso constraint in the present model, the optimizer apparently softens the contact event. While impact losses are reduced, motor utilization increases, leading to higher energy costs. RWS use does not significantly alter this low-impact, high-motor-utilization approach, but nevertheless leads to a measurable increase in walking efficiency. This is contrary to the argument that the RWS primarily acts to reduce impact losses at touchdown.

C. Control Flexibility Hypothesis

The third hypothesis states that the added control flexibility associated with the RWS allows the optimizer to plot a trajectory through the continuous phase of the state space that has less motor utilization and thus, lower energy consumption. While the stored energy in the RWS changes with wheel design, the stored momentum is a function of the torque profile only. Figure 6 shows that the RWS torque is applied at the same point in the gait cycle for the range of speeds from 0.1-0.75NS. RWS utilization is largest near touchdown, with utilization after mid-stance also becoming large at higher speeds. The heavy use surrounding touchdown naively suggests some form of contact modification, along the lines of the push-off hypothesis. The utilization, however, is also aligned with the acceleration and deceleration of the swing leg.

Centroidal dynamics are useful in analyzing complex multi-body systems [27]. Centroidal angular momentum (CAM) in particular has been examined in human walking [28] and has served as a control reference for humanoids [29], [30]. When computed for the gaits in this work, CAM exhibits the standard cyclic behavior seen in human walking [28]. It reaches a maximum positive value near mid-stance when the swing leg is moving forward and a maximum negative value at touchdown. Examination of the RWS-utilizing gaits reveals that the RWS operates in opposition to the rest of the body (Figure 7), thereby improving CAM regulation. This is verified by the RWS-utilizing model exhibiting a total CAM amplitude only 70-90% of that of the base model over most of the speed range. While improved CAM regulation has been linked to improved stability, it has not previously been associated with improved energy efficiency. Since changes in the CAM must
ultimately result from contact forces that generate moments about the centroid, a reduction in the change in CAM would directly change the torques in the stance leg motors responsible for creating contact forces. This supports the control flexibility hypothesis, and the failures of the other two hypotheses leaves it as the only viable explanation for the behavior of the RWS.

IV. Gait Simulation and Experimental Validation

While the optimization results indicate potential improvements in walking efficiency with RWS use, it is unclear to what extent these energetic savings transfer when executed on real hardware. A physics-based simulation was developed to examine the stability and efficiency of the generated gaits under closed-loop control prior to validation in hardware.

A. Control Approach

The ERNIE biped was originally designed to be controlled using the Hybrid Zero Dynamics (HZD) control approach [31] that introduces a set of virtual holonomic joint constraints in terms of a vector output function which is driven to zero under control action (typically PD control). The standard way that constraints have been formulated is with 6th-order Bézier polynomials in a measure of gait progression s, where s = 0 corresponds to the instant after touchdown and s = 1 corresponds to the moment before liftoff. The angle θ of an imaginary line between the hip and the stance foot is selected as the unactuated coordinate (or phase variable), which is used to calculate s. Control via virtual constraints of this form has consistently produced stable walking in ERNIE [32]. [33]. Experimental work [18] frequently added a fixed angular offset to both hip joint profiles as a means of achieving stable walking and adjusting walking velocity. Control of this offset can be used to achieve closed-loop velocity regulation [34], which was the case in this work.

The waypoints produced by the direct optimization must be translated into a form suitable for control. While techniques for modifying the HZD formulation to accommodate point-wise data exist [35], this work generates standard 6th-order Bézier polynomials via least squares curve fitting so as to leverage existing control software. This resulted in some minor approximation error because the polynomials are of relatively low order compared to the high dimensional representations from direct transcription.

B. Physics Model

The physics model consists of a simple forward-Euler integration of Equation 1. Position and velocity variables are generated from time-integration over the previous time step, and control variables are determined by the virtual constraint controller. Following the procedure outlined in [36], contact forces λ are found by solving the quadratic problem

$$\text{minimize} \quad \frac{1}{2} \lambda^T A \lambda + \lambda^T c$$

subject to positive normal force and friction cone constraints, where

$$A^{-1} = (J^T M^{-1} J)^{-1}$$

is the contact space inertia matrix and

$$c = J^T (\dot{q} + hM^{-1} (Bu - C\dot{q} - G))/h.$$  

To improve the smoothness and realism of the contact forces, this work uses the modification in [37] to allow inverse dynamics trajectory optimization. The A matrix is modified with the addition of a regularization matrix R, chosen to be a diagonal matrix defined by

$$R_{i,i} = ke^{d_i/k},$$

where di is the contact distance and k is a non-physical contact smoothing parameter inversely related to contact stiffness. The value corresponding to a stiff contact that yielded realistic results for the ERNIE simulation was $k = 0.0005$.

C. Simulation Results

Simulations were performed to predict the transferability of RWS-related energy results to ERNIE under high-gain PD
control. For both the base and the RWS-utilizing models, 7 gaits were selected with NSL fixed at 0.48 (0.35m) and speeds from 0.15 to 0.375 NS (0.4 m/s to 1.0 m/s) because ERNIE is known to walk reliably in this range.

Although the simulation and optimization models were identical, simulated gaits did not walk at their designed velocities. This is likely due to Bezier matching errors, imperfect joint tracking, and/or a lack of any guarantee of a stable limit cycle at the designed velocity. As is common in experiment, this was easily rectified with small torso offsets, leading to stable periodic walking at the design speeds. A simple heuristic orbit stabilizing controller was used to obtain even better speed matching to ensure the closest possible energetic correspondence between the optimized reference and the simulated result. The behavior of this controller can be observed in the video attachment.

Figure 8 compares the destination of all electrical energy that enters the motors for base gaits and RWS-utilizing gaits at 0.225 NS. Ohmic losses, though small in the optimized gaits, account for approximately 50% of total energy consumption in simulation. Negative work also increased in simulation, leading to higher regeneration losses. Meanwhile, impact losses are slightly reduced in simulation, suggesting a transfer of energy dissipation from the ground contacts to the motors. These results indicate that the method of control and gait execution can have a dramatic effect on the resulting walking efficiency. While the optimizer is free to specify joint torques with impractical precision, the feedback controller is less precise and will routinely over/undershoot its reference target, resulting in excessive torque and power. This same trend in increasing energy cost was observed in previous studies of ERNIE [32]. While impractical on ERNIE, feedforward approaches and better system modeling would likely improve the efficiency. Figure 9 shows the change in energy flow between non-RWS and RWS-utilizing gaits. Changes in impact losses are mixed, showing no consistent increase/decrease in either simulation or optimization. The most consistent trends are a decrease in ohmic losses and an increase in regenerated energy. This regeneration increase corresponds to the increase in ‘spin-up’ energy needed by the RWS to exert larger body torques. This leaves the reduction in ohmic losses as the most consistent driver of increased walking efficiency.
D. Experimental Results

The ERNIE robot was modified to include a custom RWS, built around an Allied Motion motor (MF0150010), capable of meeting the requirements observed in previous work [16] without gearing. The direct drive configuration maximizes high speed efficiency and torque bandwidth. The motor drives a modular reaction wheel whose weight and inertia can be changed by adding/removing weights. All optimization, simulation, and experimental results herein use a 2 weight configuration for a total RWS weight of 4.26 kg and a wheel inertia of 0.035 kg-m². While ERNIE is very similar to the five-link planar model, there are some important differences. ERNIE’s frontal plane dynamics and walking path are constrained by a 2.88 m long boom. A large electronics platform is attached to this boom and adds some inertia to the system in the horizontal direction. Due to the 0.3429 m offset between the legs, the ground speed of ERNIE’s hip changes for each leg, leading to left-right walking asymmetry. For consistency, the results in this section exclusively compare steps for which the inner leg is the stance leg. Experiments were based on the same 0.48 NSL gait series used in Section III-C. For each of the 14 gaits, the torso offset was adjusted by hand until the walking speed roughly matched the design speed. Data were recorded for one continuous lap of walking and then split into discrete steps. Three of these gaits are shown in the video attachment. Motor speed and torque data were used to determine the CoT of each step. Figure 10 shows the steps from these 14 experiments represented as small circles. Black lines show 2nd-order polynomial fits for the base and the RWS-utilizing ERNIE with $R^2$ values of 0.90 and 0.89, respectively. The improvement in CoT was estimated by comparing these two curve fits. This comparison is shown in Figure 11 along with optimization and simulation comparisons for the same 14 gaits.

Fig. 10: Experimental CoT of inside-leg-stance steps for the base and RWS-utilizing gaits. RWS-utilizing gaits exhibit reduced CoT over the tested range. Black lines are 2nd-order polynomial fits used for comparison in Figure 11.

The experimental improvement curve shows a significant benefit across the entire tested range and shares the same general shape as the curves from both optimization and simulation. The RWS provides the least efficiency benefit at speeds slightly above the optimal walking speed and provides an increasing efficiency benefit elsewhere. The only feature that is not consistent with simulation and optimization is the magnitude of the efficiency improvement, which appears to be about 15% larger in experiment. Visual observations and examinations of the measured data indicate some unmodeled dynamic interactions between ERNIE, the boom, and the electronics platform, which may account for this difference. Furthermore, a rough estimate of mechanical losses based on the calculated kinetic energy and the work done by the motors shows that RWS-utilizing gaits have much smaller mechanical losses than non-RWS gaits. This is not consistent with the simulation data, suggesting that this change in dissipation might be attributable to the unmodeled boom dynamics.

V. CONCLUSION

This work explored the effects of adding a high-efficiency reaction wheel system (RWS) to a planar 5-link biped with electrical actuation. All evidence suggests that with proper use, the RWS can improve walking efficiency, as measured by specific cost of transport (CoT), across a broad range of speeds and step lengths. This was predicted through both optimization and simulation and validated experimentally by equipping the ERNIE biped with an RWS. Simulation and optimization showed efficiency improvements under typical walking conditions of 5-15%, depending on postural constraints, speed, and step length. Efficiency gains in experiment were between 15% and 25%, with the difference attributed to differences between the idealized model and the practical experimental system.

This work sought to explain the mechanism of efficiency improvement in terms of three energy saving concepts: kinetic energy storage and return (ESR), impact loss reduction, and improved control flexibility. While the RWS can naturally be viewed as an ESR system, this functionality was shown to be mostly incidental. Control moment gyroscopes (CMGs), which do not significantly alter their stored kinetic energy, would...
be similarly effective and possibly preferable due to lower electrical demand. RWS use primarily before and after touchdown suggests impact modification, but RWS use was not associated with a consistent reduction in impact loss, and, in some cases, RWS-utilizing gaits exhibited larger impact losses. This demonstrates that walking cost and impact loss are not directly related when actuator efficiency is considered. Instead, the RWS offloads the leg motors, leading to reduced ohmic loss in the RWS-utilizing biped. The RWS accomplishes this by providing some cancelation of swing leg momentum and improving centroidal momentum regulation.

Because the efficiency-improving behavior of the RWS emerges from the holistic combination of kinematics, dynamics, and motor parameters of the system on which it operates, generalizing its potential benefits to other systems is difficult. Both the magnitude of efficiency gains and the specifics of RWS use will change with these factors, necessitating full body optimization to quantify. The biped in this work has point feet and a relatively high ohmic loss coefficient, meaning that these results may not be applicable to heavily geared humanoid robots. Nevertheless, the concept of high efficiency inertial actuators improving the efficiency of walking systems has been validated and should be transferable to other robots.

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REFERENCES